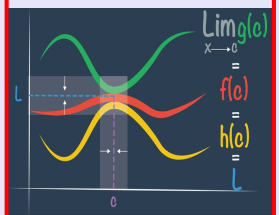
Calculus I

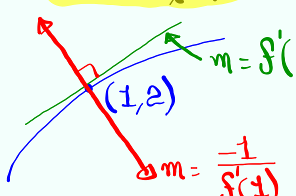
Lecture 25



Feb 19-8:47 AM

Class Quiz 10

Find the equation of the normal line to the graph of $f(x) = \frac{3x+1}{x^2+1}$ at the point $(1,2)$ ✓



$$f(1) = \frac{3(1)+1}{1^2+1} = \frac{4}{2} = 2$$

$$f'(x) = \frac{3(x^2+1) - (3x+1)\cdot 2x}{(x^2+1)^2}$$

$$f'(x) = \frac{3x^2+3-6x^2-2x}{(x^2+1)^2}$$

$$f'(x) = \frac{-3x^2-2x+3}{(x^2+1)^2}$$

$$f'(1) = \frac{-3-2+3}{(1+1)^2} = \frac{-2}{4} = \frac{-1}{2}$$

$$m = \frac{-1}{\frac{-1}{2}} = 2$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = 2(x - 1)$$

$$\boxed{y = 2x}$$

Oct 10-6:52 AM

Chain Rule

If $F(x) = f(g(x))$, then

$$F'(x) = f'(g(x)) \cdot g'(x)$$

If $y = f(u)$ and $u = g(x)$, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Ex: $y = (x^2 + 8)^3$

Let $u = x^2 + 8$

$y = u^3$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= 3u^2 \cdot 2x$$

$$y' = \frac{dy}{dx} = 3(x^2+8)^2 \cdot 2x$$

$$\boxed{y' = 6x(x^2+8)^2}$$

Oct 9-8:34 AM

$$f(x) = \sin(x^2 - 2x)$$

$$y = \sin(\underbrace{x^2 - 2x})$$

$u = x^2 - 2x$

$y = \sin u$

$$y' = \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$y' = \cos u \cdot (2x - 2)$$

$$y' = \cos(x^2 - 2x) \cdot (2x - 2)$$

$$y' = (2x-2) \cdot \cos(x^2 - 2x)$$

Oct 10-7:51 AM

$f(x) = \tan(\sqrt{x})$

$f'(x) = \sec^2(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$

$= \frac{1}{2\sqrt{x}} \sec^2(\sqrt{x})$

$\frac{d}{dx}[\sqrt{x}] =$

$\frac{d}{dx}[x^{1/2}] =$

$\frac{1}{2} \cdot x^{\frac{1}{2}-1} =$

$\frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$

Given $f(x) = \cos(\sin x^2)$

Find $f'(1)$

$f'(x) = -\sin(\sin x^2) \cdot \cos x^2 \cdot 2x$

$f'(1) = -\sin(\sin 1^2) \cdot \cos 1 \cdot 2 \cdot 1$

$\boxed{f'(1) = -2\sin(\sin 1) \cdot \cos 1}$

Oct 10-7:54 AM

Find slope of the tan. line to the graph of $f(x) = (4x - x^2)^{100}$ at $x = 0$.

$f'(x) = 100(4x - x^2)^{99} \cdot (4 - 2x)$

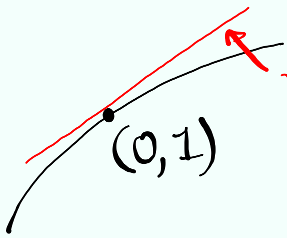
$m_{\text{tan. line at } x=0} = f'(0) = 100(4 \cdot 0 - 0^2)^{99} \cdot (4 - 2 \cdot 0)$

$= 100 \cdot 0^{99} \cdot 4 = 400 \cdot 0 = \boxed{0}$

Oct 10-8:04 AM

Find eqn of tan. line to the graph of

$f(x) = \cos x^2$ at $x = 0$.



$m = f'(0)$

$= 0$

$f(x) = \cos x^2$

$f'(x) = -\sin x^2 \cdot 2x$

$f'(0) = -\sin 0^2 \cdot 2(0)$

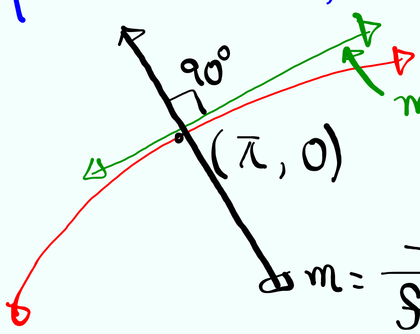
$= 0$

$y - 1 = 0(x - 0)$

$\boxed{y = 1}$

Oct 10-8:07 AM

Find eqn of the normal line to the graph of $f(x) = \sin(\sin x)$ at $x = \pi$.



$m = f'(\pi)$

$m = \dfrac{-1}{f'(\pi)}$

$m = \dfrac{-1}{-1} = 1$

$f(x) = \sin(\sin x)$

$f'(x) = \cos(\sin x) \cdot \cos x$

$f'(\pi) = \cos(\sin \pi) \cdot \cos \pi$

$= \cos 0 \cdot (-1)$

$= 1 \cdot (-1)$

$= -1$

$y - 0 = 1(x - \pi) \rightarrow \boxed{y = x - \pi}$

Oct 10-8:14 AM

$f(t) = \sqrt{\dfrac{t}{t^2+4}}$ find $f'(t)$

$f(t) = \left[\dfrac{t}{t^2+4}\right]^{1/2}$

t^2+4-2t^2

$f'(t) = \frac{1}{2}\left[\dfrac{t}{t^2+4}\right]^{\frac{1}{2}-1} \cdot \dfrac{1(t^2+4)-t\cdot 2t}{(t^2+4)^2}$

$\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^{n}$

$f'(t) = \frac{1}{2}\cdot\left(\dfrac{t}{t^2+4}\right)^{-\frac{1}{2}} \cdot \dfrac{4-t^2}{(t^2+4)^2}$

$\dfrac{x^m}{x^n} = x^{m-n}$

$= \frac{1}{2}\cdot\left(\dfrac{t^2+4}{t}\right)^{1/2} \cdot \dfrac{4-t^2}{(t^2+4)^2}$

$= \frac{1}{2}\cdot\dfrac{(t^2+4)^{1/2}}{\sqrt{t}} \cdot \dfrac{4-t^2}{(t^2+4)^2} = \dfrac{4-t^2}{2\sqrt{t}\,(t^2+4)^{3/2}}$

Oct 10-8:23 AM

$y = \cot^2(\sin x)$ find $\dfrac{dy}{dx}$

$y = \left[\cot(\sin x)\right]^{2}$

$y' = 2\left[\cot(\sin x)\right]^{1}\cdot\left(-\csc^2(\sin x)\right)\cdot\cos x$

$= -2\cos x \cdot \cot(\sin x)\cdot \csc^2(\sin x)$

Oct 10-8:31 AM